POISSON PROCESS IN R

GROUP 6 :

TOPIC NO. 3

a.  Run the process given parameters  
b.  Given number of events in the given time, find out the expected time   
c.  simulate thinning process  
d.  simulate super position of the process.  
e. simulate compound process - similar to exam question of the number of people in restaurants   
f. simulate non homegeneous Poisson process

a. to simulate 100 random numbers from poisson process with lambda = 7, also plotted poisson distribution

set.seed(456)

x<-rpois(100,7)

x

plot(x,dpois(x,7))

plot

b) Let the given number of events be 5 per minute , to find the expected time

c) Assume 20 people arrive at a hospital in 1 hour which follows poisson distribution.10% of them get hospitalized and 90% of them do not get hospitalized. (a)What is the expected time for 29th hospitalized patient and 29th non-hospitalized patient to enter? (b)what is probability that there will be 17 patients hospitalized in 7 hours ? © Between 7th and 11th hour , if there are 72 patients, what is the probability that 15 of them are hospitalized ?

Solution

# lambda = 20 , hospitalized lambda(lh) = 2 , non-hospitalized lambda(nlh) = 18

# 1st part : expected time for 29 hospitalized patients(th) and 29 non-hospitalized(tnh) patients

lh <- 2

lnh <- 18

th <- 29/lh

tnh <- 29/lnh

# 2nd part : p(x=50) in 7 hours : lh = 2 , t = 7 , lht = 14, x= 17 , lambda = 14

dpois(17,14)

## [1] 0.0712829

# 3rd part

(dpois(15,8)\*dpois(67,72))/dpois(72,80)

## [1] 0.01184937

d) Customers arrive at a store at the rate of 10 per hour.Each is either a male or female with prob 0.5.Assume that you know that exactly 10 women entered within some hr

a)computer the prob that exactly 10 men also entered

B)compute the prob that at least 20 customers have entered

Solution

#lamba = 10

#Let X=Arriving customer is a male, Y= Arriving customer is a female

#We are given that arriving customer is either male or female with probability 1/2

#i.e.P(x=y)=1/2

#Also since X and Y are independent poisson processes so rate=(lambda x t)=(1/2 x 10)=5

#a)P(X=10)

dpois(10,5)

#b)P(k men entered)

x=1-dpois(10,5)\*dpois(11,5)\*dpois(12,5)\*dpois(13,5)\*dpois(14,5)\*dpois(15,5)\*dpois(16,5)\*dpois(17,5)\*dpois(18,5)\*dpois(19,5)\*dpois(20,5)

x

Answers:a)0.01813279, b)1

e) Number of claims to insurance company as Poisson process of 5 claims per day and the claim amount is uniformly distributed within range of 10000-50000. What is the expected amount and variance of the claim amount?

Solution

> # parameters for uniform distribution.

> a= 10000

> b= 50000

> u.s=runif(n=100,min = a,max=b)

> mean.x=mean(u.s)

> mean.x

[1] 31668.6

> e.x= var(u.s)+(mean(u.s)\*\*2)

> e.x

[1] 1125054423

> # parameter for Poisson distribution.

> lambda = 5

> t=31

> #Sample mean and variance

> (mean.s= lambda\*t\*mean.x)

[1] 4908633

> (var.s= lambda\*t\*e.x)

[1] 174383435544

> # theoretical mean and variance:

> (mean.t = lambda \* t \* (b+a)/2)

[1] 4650000

> (var.t = lambda\*t\*(a\*2+b\*2+a\*b)/3)

[1] 160166666667

f)

Consider a Poisson process gif.latex (54×20), with non-homogeneous intensity http://latex.codecogs.com/gif.latex?%5Clambda(t). Here, we consider a deterministic function, not a stochastic intensity. Define the cumulated intensity

http://latex.codecogs.com/gif.latex?%5CLambda(t)=%5Cint_0%5Et%5Clambda(s)ds

in the sense that the number of events that occurred between time gif.latex (8×13) and gif.latex (6×12) is a random variable that is Poisson distributed with parameter  http://latex.codecogs.com/gif.latex?%5CLambda(t).

For example, consider here a cyclical Poisson process, with intensity

To compute the cumulated intensity, consider a very general function

The idea is to generate a Poisson process on a finite interval http://latex.codecogs.com/gif.latex?%5B0,T%5D.

The first code is based on a proposition from Çinlar (1975),

1. start with http://latex.codecogs.com/gif.latex?s=0
2. generate gif.latex (96×19)
3. set gif.latex (112×19)
4. set gif.latex (6×12) denote gif.latex (124×19)
5. deliver http://latex.codecogs.com/gif.latex?t
6. go to step 2.

In order to get the infinimum of gif.latex (12×13), consider a code as

lambda=function(x) 100\*(sin(x\*pi)+1)  
Lambda=function(t) integrate(f=lambda,lower=0,upper=t)$value  
Tmax=5  
s=0; v=seq(0,length=10)  
X=numeric(10)  
while(X[length(X)]<=Tmax){  
 u=runif(1)  
 s=s-log(u)  
 t=min(v[which(Vectorize(Lambda)(v)>=s)])  
 X=c(X,t)  
}  
hist(X,breaks=seq(0,max(X)+1,by=.1),col="yellow")  
u=seq(0,max(X),by=.02)  
lines(u,lambda(u)/10,lwd=2,col="red")

